Some Quality Measures for Fuzzy Association Rules

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Abstract. Several approaches generalizing crisp association rules to fuzzy association rules have been proposed. In an our previous paper we introduced a pair of confidence measures for crisp association rules from which one can be obtained the majority known quality measures. In this paper, starting from these results we give an extension to fuzzy association rules.
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1 INTRODUCTION

Data mining, also known as knowledge discovery in databases, has as goal to extract higher level information from a lot of row data. Mining association rules is one of the most important task in data mining.

Association rules provide a way to obtain certain dependencies between attributes in a database. Usually, an association rule work with a database $\mathcal{D}$ containing records described by binary attributes values. Each record $x \in \mathcal{D}$ coresponds to a transaction and for each attribute A, $A(x)$ is either 1 or 0 indicating whether or not item A was bought in transaction x. More general, an attribute value can be a a number from the interval $[0, 1]$

An association rule is an expression with following form $A \rightharpoonup B$, where A and B are attributes, or sets of attributes. The meaning of a rule $A \rightharpoonup B$ is that when A is bought in a transaction, B is likely to be bought as well. In order to express the quality of an association rule one uses measures such as [1]:

- the **support** = the number of transactions in which both A and B where bought
- the **confidence** = the percentage of transactions containing A that contain B as well.

In most applications from real life, databases contain other attribute values besides 0 and 1. For instance, the attribute $profit$ can take values as *low*, *medium*, *high* which can be represented as fuzzy sets.

Mining association rules based on fuzzy sets can handle quantitative and categorical data, providing the necessary support to use uncertain data types with existing algorithms; it can also create smoother transition boundaries between partitions for numerical values, constituting a perfect solution for both well-defined and imprecise data.

Association rules can be rated by a number of quality measures, among which *support* and *confidence* measures are essential ones. The support measure quantifies the statistical significance of a rule $A \rightharpoonup B$ i.e. what is the degree in which the association rule corresponds with the results of statistical analysis of the transaction database, while confidence measure quantify the rule strength i.e. the degree of trust of the rule. The basic problem of mining association rules is then to generate all association rules $A \rightharpoonup B$ that have support and confidence greater than user-defined thresholds. These measures can be generalized for fuzzy association rules in several ways. The goal of this paper is not to introduce yet another series of quality measures, but to offer a basic extension usable in the case of fuzzy association rules.

The next section describes the support and confidence measures in the case of crisp association rules, including the standard classification of transactions in a database as positive or negative examples of an association rule between attributes.

Section 3 is devoted to the "fuzzy case" of association rules. We recall some basic concepts of fuzzy sets, fuzzy sets operations and the definition of fuzzy association rules. This framework permits us to define the support and confidence measures for fuzzy association rules. As in the crisp case we distinguish between four kinds of support measures and two kinds of confidence measures. The confidence measures $fconf_*$ and $fconf^*$ represent a general and unifier frame for the quality measures, because majority of them can be obtained particularizing the working framework i.e. by specifying the t-norm, t-conorm and negation we use, the thresholds α and β, and by using different kinds of support measures as values for parameters M_1 and M_2.

The Conclusion section come with the justification of the new introduced measures as being more general than these introduced in [7]. Finally, the future work is concerned to research for criteria which can be used in choosing appropriate measures for domain dependent applications.

2 CRISP ASSOCIATION RULES

Let $\mathcal{D}$ be a non-empty data table containing records described by theirs values binary attributes. We will denote by D_A the set of all transactions x in which A was purchased, i.e.

$$D_A = \{x \in \mathcal{D} | A(x) = 1\}.$$

We denote by coD_A the set of transactions that not contain the item A, i.e

$$coD_A = \{x \in \mathcal{D} | A(x) = 0\}$$

We will work with simple association rules $A \rightharpoonup B$ in which A and B are both attributes and not sets of attributes; this is not a limitation since we can always introduce a new attribute combining others. The informal meaning of such a rule is "every transaction that contains A contains B too".

Definition 1 (Support). *The* support count *respectively* support *of an association rule $A \rightharpoonup B$ is usually defined as:*

$$supp\#(A \rightharpoonup B) = |D_A \cap D_B|$$

and respectively

$$supp(A \rightharpoonup B) = \frac{|D_A \cap D_B|}{|\mathcal{D}|}$$

where $|D_A|$ is the number of transactions that contain A. Support *is the percentage of transactions where the rule holds.*

This definition of support count positive examples, i.e. transactions that explicitly support the hypothesis expressed by the association rule.

De Cock et al. [3], [4] classify transactions with respect of an association rule:

Definition 2 (Transaction Classification). *Let $A \rightharpoonup B$ be an association rule and x be a transaction. Then*

- *x is a* positive example *iff* $x \in D_A \wedge x \in D_B$
- *x is a* non-positive example *iff* $x \notin D_A \vee x \notin D_B$
- *x is a* negative example *iff* $x \in D_A \wedge x \notin D_B$
- *x is a* non-negative example *iff* $x \notin D_A \vee x \in D_B$

According to this definition, different measures (of support count type) can be considered:

- (minimum)support count $minsupp\#(A \rightharpoonup B) = |D_A \cap D_B|$
- maximum opposition count: $maxopp\#(A \rightharpoonup B) = |coD_A \cup coD_B|$
- minimum opposition count: $minopp\#(A \rightharpoonup B) = |D_A \cap coD_B|$
- maximum support count: $maxsupp\#(A \rightharpoonup B) = |coD_A \cup D_B|$

and correspondingly (measures of support type):

- (minimum) support: $minsupp(A \rightharpoonup B) = |D_A \cap D_B|/|\mathcal{D}|$
- maximum opposition: $maxopp(A \rightharpoonup B) = |coD_A \cup coD_B|/|\mathcal{D}|$
- minimum opposition: $minopp(A \rightharpoonup B) = |D_A \cap coD_B|/|\mathcal{D}|$
- maximum support: $maxsupp(A \rightharpoonup B) = |coD_A \cup D_B|/|\mathcal{D}|$

The classical confidence is usually defined as below:

Definition 3 (Confidence).

$$conf(A \rightharpoonup B) = \frac{supp\#(A \rightharpoonup B)}{supp\#(A)} = \frac{|D_A \cap D_B|}{|D_A|} = \\ = \frac{supp\#(A \rightharpoonup B)}{supp\#(A \rightharpoonup B) + supp\#(A \rightharpoonup coB)} \tag{1}$$

Confidence *is the conditional probability of* B *with respect to* A *or, in other words, the relative cardinality of* B *with respect to* A.

Hullermeier [6] was proposed another definition, *n-confidence*:

$$conf_n(A \rightharpoonup B) = \frac{supp\#(A \rightharpoonup B)}{supp\#(A \rightharpoonup coB)} = \frac{minsupp\#(A \rightharpoonup B)}{minopp\#(A \rightharpoonup B)} \tag{2}$$

and De Cook et al. in [3] introduce two new measures of confidence, namely p-confidence and o-confidence (pessimistic and optimistic confidence):

$$conf_p(A \rightharpoonup B) = \frac{supp\#(A \rightharpoonup B)}{maxopp\#(A \rightharpoonup B)} \tag{3}$$

$$conf_o(A \rightharpoonup B) = \frac{maxsupp\#(A \rightharpoonup B)}{minopp\#(A \rightharpoonup B)} \tag{4}$$

with the property:

$$conf_p(A \rightharpoonup B) \leq conf_n(A \rightharpoonup B) \leq conf_o(A \rightharpoonup B)$$

Starting from a pair (M_1, M_2) of quality measures (i.e support or confidence measures) for association rules, with the following property:

$$M_1(A \rightharpoonup B) \leq M_2(A \rightharpoonup B)$$

two new confidence measures were proposed in [7]:

a) *inferior confidence*

$$conf_*(A \rightharpoonup B) = \frac{\alpha \cdot M_1(A \rightharpoonup B)}{(1-\beta) \cdot M_1(A \rightharpoonup B) + \beta \cdot M_2(A \rightharpoonup coB)} \tag{5}$$

and

b) *supperior confidence*

$$conf^*(A \rightharpoonup B) = \frac{\alpha \cdot M_2(A \rightharpoonup B)}{(1-\beta) \cdot M_2(A \rightharpoonup B) + \beta \cdot M_1(A \rightharpoonup coB)} \tag{6}$$

with $\alpha, \beta \in [0, 1]$.

The following relation is straightforward:

$$conf_*(A \rightharpoonup B) \leq conf^*(A \rightharpoonup B)$$

and for $\alpha + \beta \leq 1$ we have $conf_*(A \rightharpoonup B), conf^*(A \rightharpoonup B) \in [0, 1]$.

Majority of classic quality measures are obtained from $conf_*$ and $conf^*$ particularizing α, β, M_1 and M_2 .

3 FUZZY ASSOCIATION RULES

Recall some basic notions about fuzzy set operations. The notions of complement, intersections and union for fuzzy sets are defined by means of negator, t-norm and t-conorm operators.

A fuzzy set A in X is an $X \to [0,1]$ mapping. An increasing, associative and commutative $[0,1]^2 \to [0,1]$ mapping is called *t-norm* $\mathcal{T}$ if it satisfies $\mathcal{T}(x,1) = x$ for all x in $[0,1]$ and a *t-conorm* $\mathcal{S}$ if it satisfies $\mathcal{S}(x,0) = x$ for all x in $[0,1]$. A *negator* $\mathcal{N}$ is a decreasing $[0,1] \to [0,1]$ mapping satisfying $\mathcal{N}(0) = 1$ and $\mathcal{N}(1) = 0$.

For A and B fuzzy sets in X, the complementation, intersection and union can be defined respectively, by

$$co_{\mathcal{N}}A(x) \doteq \widetilde{A}(x) = \mathcal{N}(A(x)),$$

$$(A \cap_{\mathcal{T}} B)(x) = \mathcal{T}(A(x), B(x)),$$

$$(A \cup_{\mathcal{S}} B)(x) = \mathcal{S}(A(x), B(x))$$

for all $x \in X$.
The cardinality of a fuzzy set A in X is defined as

$$|A| = \sum_{x \in X} A(x).$$

The formal definition of fuzzy association rules as in [2] is the following:

Let $\mathcal{D} = \{t_1, \ldots, t_n\}$ a transactional database. We consider that this database is characterized by a set of categorical or quantitative attributes (items).

Let $\mathcal{I} = \{i_1, \ldots, i_m\}$ the set of these attributes. For each attribute i_k, $(k = 1, \ldots, m)$ we will consider $n(k)$ associated fuzzy sets. Let $F_{i_k} = \{F^1_{i_k}, \ldots, F^{n(k)}_{i_k}\}$ be the set of all these fuzzy sets.

For an attribute i_k and a fuzzy set $F^j_{i_k}$, the membership function is $\mu_{F^j_{i_k}}$. Therefore we have:

$$\mu_{F^j_{i_k}} : dom(i_k) \to [0,1],\ k = 1, \ldots, m, j = 1, \ldots, n(k)$$

For ease of notation we use the same expression F_A to denote a fuzzy set associated with the attribute A and the membership function μ_{F_A} associated to the fuzzy set F_A. For a specified transaction $t \in \mathcal{D}$ we can retrieve the value of attribute i_k using $t[i_k]$.

Example 1. In Table 1, we illustrate a database sample with quantitative attributes.
Here, we have $\mathcal{D} = \{t_1, t_2, t_3, t_4, t_5\}$, and $\mathcal{I} = \{Age,\ Income,\ Cars\}$. If we want to know the value of $Cars$ for the third record, we can use $t_3[Cars]$ and obtain 2.

For example, we can take into consideration for the attribute Age the following three fuzzy sets: "*young*", "*middle*" and "*old*"; we have $F_{Age} =$

TID	Age	Income	Cars
1	22	2000	0
2	30	4000	1
3	30	5000	2
4	40	6000	1
5	45	4000	1

Table 1. Database sample

$\{young,\ middle,\ old\}$. If we consider that a person is "young" if his age is between 25 and 30 years old, then the membership function of the fuzzy set "*young*" can be defined as:

$$\mu_{young}\colon [0,100] \to [0,1],$$

$$\mu_{young}(x) = \begin{cases} 0, & \text{if } x < 20 \\ \frac{(x-20)}{(25-20)}, & \text{if } 20 \le x < 25 \\ 1, & \text{if } 25 \le x \le 30 \\ \frac{(35-x)}{(35-30)}, & \text{if } 30 < x \le 35 \\ 0, & \text{if } x > 35 \end{cases}$$

Then, $\mu_{young}(t_1) = \mu_{young}(t_1[age]) = \mu_{young}(22) = 0.4$.

Definition 4. *A **fuzzy association rule** is an implication with following form*

$$X \in F_X \Rightarrow Y \in F_Y$$

where $X, Y \subset \mathcal{I}$, $X \cap Y = \emptyset$, $X = \{x_1, \ldots, x_p\}$, $Y = \{y_1, \ldots, y_q\}$. $F_X = \{a_1, \ldots, a_p\}$ *and* $F_Y = \{b_1, \ldots, b_q\}$ *are fuzzy sets related to attributes from* X, *respectively* Y. *More exactly,* $a_i \in F_{x_i}$, $(i = 1, \ldots, p)$, *and* $b_i \in F_{y_i}$, $(i = 1, \ldots, q)$.
We denote this rule with

$$\langle X, F_X \rangle \rightharpoonup \langle Y, F_Y \rangle$$

The intuitively signification of this fuzzy association rule $\langle X, F_X \rangle \rightharpoonup \langle Y, F_Y \rangle$ is: "if a transaction (tuple) satisfies the property $X \in F_X$ then it will satisfy the property $Y \in F_Y$ with a high probability also".

Example 2. An example of a fuzzy association rule for the database given in Example 1 is "If Age is young and Income is high then Cars is many". Here $X = \{Age, Income\}$, $Y = \{Cars\}$, $F_X = \{young, high\}$, $F_Y = \{many\}$, and the rule can be represented as $\langle \{Age, Income\}, \{young, high\} \rangle \rightharpoonup \langle \{Cars\}, \{many\} \rangle$

We will work with simple association rules $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ in which A and B are both attributes and not sets of attributes.

Replacing $|D_A \cap D_B|$ and $|D_A \cup D_B|$ by $\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} F_B)(x)$ and respectively by $\sum_{x\in\mathcal{D}}(F_A \cup_{\mathcal{S}} F_B)(x)$ in the formulas associated to a crisp rule $A \rightharpoonup B$ we obtain the fuzzy version for support and confidence measures associated to a fuzzy association rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$.

Definition 5 (Fuzzy Support). *The* fuzzy support count *respectively* fuzzy support *of a fuzzy association rule $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ is usually defined as:*

$$fsupp\#(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} F_B)(x)$$

and respectively

$$fsupp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} F_B)(x)}{|\mathcal{D}|}$$

We can also extend the other support measures defined for the crisp association rules:

Definition 6. *Let $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ a fuzzy association rule. Then, we define:*

a) fuzzy (minimum) support:

$$fminsupp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} F_B)(x)}{|\mathcal{D}|}$$

b) fuzzy maximum opposition:

$$fmaxopp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in\mathcal{D}}(\widetilde{F}_A \cup_{\mathcal{S}} \widetilde{F}_B)(x)}{|\mathcal{D}|}$$

c) fuzzy minimum opposition:

$$fminopp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} \widetilde{F}_B)(x)}{|\mathcal{D}|}$$

d) fuzzy maximum support:

$$fmaxsupp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in\mathcal{D}}(\widetilde{F}_A \cup_{\mathcal{S}} F_B)(x)}{|\mathcal{D}|}$$

Similarly, we can define the corresponding count measures: $fminsupp\#$, $fmaxopp\#$, $fminopp\#$, $fmaxsupp\#$.

Definition 7 (Fuzzy Confidence). *Let $\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle$ a fuzzy association rule. The* fuzzy confidence *of rule is defined as:*

$$fconf(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} F_B)(x)}{\sum_{x\in\mathcal{D}} F_A(x)}$$

The inferior confidence ($conf_*$) and superior confidence ($conf^*$) defined for crisp association rules can be rewritten for fuzzy association rules as:

i) *fuzzy inferior confidence*: $fconf_*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) =$

$$= \frac{\alpha \cdot M_1(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{(1-\beta)\cdot M_1(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) + \beta \cdot M_2(\langle A, F_A\rangle \rightharpoonup \langle B, \widetilde{F}_B\rangle)} \tag{7}$$

and

ii) *fuzzy superior confidence*: $conf^*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) =$

$$= \frac{\alpha \cdot M_2(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)}{(1-\beta)\cdot M_2(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) + \beta \cdot M_1(\langle A, F_A\rangle \rightharpoonup \langle B, \widetilde{F}_B\rangle)} \tag{8}$$

with $\alpha, \beta \in [0, 1]$.

The measures $fconf_*$ and $fconf^*$ represent a general and unifier frame for the quality measures, because majority of them can be obtained particularizing $\mathcal{T}$, $\mathcal{S}$, $\mathcal{N}$, α, β, M_1 and M_2. For instance:

a) for $M_1 = M_2 = fsupp\#$, $\alpha = \beta = \frac{1}{2}$, $\mathcal{T}(a, b) = ab$ and $\mathcal{N}(a) = 1 - a$ we have:

$$fconf_* = fconf^* = fconf$$

b) for $M_1 = M_2 = fsupp\#$ and $\alpha = \beta = 1$ we have:

$$fconf_* = fconf^* = fconf_n$$

$$fconf(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) = \frac{\sum\limits_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} F_B)(x)}{\sum\limits_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} \widetilde{F}_B)(x)}$$

c) for $M_1 = fminsupp\#$, $M_2 = fmaxsupp\#$ and $\alpha = \beta = 1$ we obtain

$$\begin{aligned}
fconf_*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) &= fconf_p(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)\\
&= \frac{\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} F_B)(x)}{\sum_{x\in\mathcal{D}}(\widetilde{F}_A \cup_{\mathcal{S}} \widetilde{F}_B)(x)}\\
fconf^*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) &= fconf_o(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle)\\
&= \frac{\sum_{x\in\mathcal{D}}(\widetilde{F}_A \cup_{\mathcal{S}} F_B)(x)}{\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} \widetilde{F}_B)(x)}
\end{aligned}$$

d) for $M_1 = fminsupp\#$, $M_2 = fmaxsupp\#$, $\alpha = \beta = \frac{1}{2}$ and $\mathcal{N}(a) = 1 - a$ we obtain

$$\begin{aligned}
fconf_*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) &= fminsup(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) \\
&= \frac{\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} F_B)(x)}{|\mathcal{D}|} \\
fconf^*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) &= fmaxsup(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) \\
&= \frac{\sum_{x\in\mathcal{D}}(\widetilde{F}_A \cup_{\mathcal{S}} F_B)(x)}{|\mathcal{D}|}
\end{aligned}$$

e) for $M_1 = fminopp\#$, $M_2 = fmaxopp\#$, $\alpha = \beta = \frac{1}{2}$ and $\mathcal{N}(a) = 1 - a$ we obtain

$$\begin{aligned}
fconf_*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) &= fminopp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) \\
&= \frac{\sum_{x\in\mathcal{D}}(F_A \cap_{\mathcal{T}} \widetilde{F}_B)(x)}{|\mathcal{D}|} \\
fconf^*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) &= fmaxopp(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) \\
&= \frac{\sum_{x\in\mathcal{D}}(\widetilde{F}_A \cup_{\mathcal{S}} \widetilde{F}_B)(x)}{|\mathcal{D}|}
\end{aligned}$$

Also, we obtain new measures:

f) for $M_1 = fconf\_p, M_2 = fconf\_o$ and $\alpha = \beta = \frac{1}{2}$ we have:

$$fconf\_po_*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) =$$
$$= \frac{\left(\sum\limits_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} F_B)(x)\right)^2}{\left(\sum\limits_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} F_B)(x)\right)^2 + \left(\sum\limits_{x\in\mathcal{D}} (\widetilde{F}_A \cup_{\mathcal{S}} \widetilde{F}_B)(x)\right)^2}$$

$$fconf\_po^*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) =$$
$$= \frac{\left(\sum\limits_{x\in\mathcal{D}} (\widetilde{F}_A \cup_{\mathcal{S}} F_B)(x)\right)^2}{\left(\sum\limits_{x\in\mathcal{D}} (\widetilde{F}_A \cup_{\mathcal{S}} F_B)(x)\right)^2 + \left(\sum\limits_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} \widetilde{F}_B)(x)\right)^2}$$

g) for $M_1 = fconf\_p$, $M_2 = fconf\_n$ (or $M_1 = fconf\_n$, $M_2 = fconf\_o$) and $\alpha = \beta = \frac{1}{2}$ we have:

$$fconf\_pn_*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) =$$

$$= \frac{\left(\sum_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} F_B)(x)\right)^2}{\left(\sum_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} F_B)(x)\right)^2 + \left(\sum_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} \widetilde{F}_B)(x)\right)\cdot\left(\sum_{x\in\mathcal{D}} (\widetilde{F}_A \cup_{\mathcal{S}} \widetilde{F}_B)(x)\right)}$$

$$fconf\_pn^*(\langle A, F_A\rangle \rightharpoonup \langle B, F_B\rangle) =$$

$$= \frac{\left(\sum_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} F_B)(x)\right)\cdot\left(\sum_{x\in\mathcal{D}} (\widetilde{F}_A \cup_{\mathcal{S}} F_B)(x)\right)}{\left(\sum_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} F_B)(x)\right)\cdot\left(\sum_{x\in\mathcal{D}} (\widetilde{F}_A \cup_{\mathcal{S}} F_B)(x)\right) + \left(\sum_{x\in\mathcal{D}} (F_A \cap_{\mathcal{T}} \widetilde{F}_B)(x)\right)^2}$$

where F_A and F_B are the fuzzy sets associated with the attribute A and B, respectively.

4 CONCLUSIONS

In this paper, some quality measures defined for crisp association rule were extended for fuzzy rules. If fuzzy set became crisp set, i.e.

$$F_A(x) = \chi_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

and $\mathcal{T}(a,b) = \min(a,b)$, $\mathcal{S}(a,b) = \max(x,y)$ and $\mathcal{N}(a) = 1 - a$ we obtain the quality measures defined in [7] for crisp association rules.

However, since on the model of fuzzy association rules is possible to define a large number of support and confidence quality measures, one of the future tasks will be to develop criteria to choose the appropriate class of measures. We shall report on our progress in subsequent papers.

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